Name: _____

418 Quiz 5

1. The differential equation

$$\begin{aligned} DE &: u_t = u_{xx} - 2u, \\ BC &: u_x(t,0) = u_x(t,1) = 0 \\ IC &: u(0,x) = \begin{cases} 4 & \text{when } 0 < x < 1/2; \\ 0 & \text{when } 1/2 < x < 1 \end{cases} \end{aligned}$$

models the diffusion of heat in a metal rod of unit length with insulated ends but that is emanating heat into the surrounding air. $-2u$ is called a dissipation term.

(a) Use $u = w(t)v(x)$ to find the separable solutions. (Hint: be careful with how different λ create different cases).

(b) Use the boundary conditions to find the general form of the separable solution.

(c) Use the initial conditions to find the coefficients using Fourier series techniques.

(d) Describe the long term behavior of the system - what is dissipation term doing to the solution as $t \to \infty$?

2. (bonus)

Consider the wave equation with a damping force

$$\begin{aligned} DE &: u_{tt} + 2u_t = u_{xx} \\ BC &: u_x(t,0) = u_x(t,1) = 0 \\ IC &: u(0,x) = 3 + 2\cos(\pi x) - \cos(2\pi x) \\ & \quad u_t(0,x) = 0 \end{aligned}$$

(a) Use $u = w(t)v(x)$ to find the separable solutions.

(b) Use the boundary conditions to find the general form of the separable solution.

(c) Use the initial conditions to find the coefficients.

(d) Describe the long term behavior of the system - what does the damping term do to the solution as $t \to \infty$?