

Name: _____

418 Quiz 4

1. Find the Fourier series expansion of $f(x) = x^2$ on the interval $(-\pi, \pi)$ using the orthogonal family $\{1, \cos(nx), \sin(nx)\}$.
2. Discuss the convergence of the Fourier series to the 2π -periodic extension of f , with particular attention to the endpoints.
3. Evaluate your series at $x = \pi$ and show that $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$. (Congrats, you've solved the famous Basel problem).
4. Find the Fourier series representation of x^3 . Hint: use integrals. you might productively use the fact that the Fourier series for x is

$$x \sim 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx).$$