418 Exam warmup

1. Given an example for each case:
   (a) A second order, linear, homogeneous PDE in three variables.
   (b) A nonlinear PDE in two variables.
   (c) A first order, inhomogeneous linear PDE in two variables.

2. Find the solution $u = u(t, x)$ to the PDE

$$\begin{aligned} u_t - 2x\, u_x &= 0 \\ u(0, x) &= \sin(x). \end{aligned}$$

3. (a) Find the characteristic change of variables for the equation

$$2u_t - u_x + u = 0.$$

   (b) Use the change of variables from part (a) to solve the PDE.

4. (a) Solve the PDE

$$u_{tt} - 2u_{xt} - 3u_{xx} = 0.$$

   (b) Use your answer to solve the initial value problem

$$\begin{aligned} u_{tt} - 2u_{xt} - 3u_{xx} &= 0, \\ u(x, 0) &= x, \\ u_t(x, 0) &= e^x. \end{aligned}$$

5. Find the eigensolutions $u = e^{\lambda t} v(x)$ to the differential equation

$$u_{tt} - 4u_{xx} = 0.$$