suppose our domain is $x \in (-\ell, \ell)$.

we can <u>rescale</u> x via $y = \frac{\pi x}{\ell}$

so that $-\ell \leq x \leq \ell \Rightarrow -\pi \leq y \leq \pi$.
let $F(y) = f(x) = f\left(\frac{\ell y}{\pi}\right)$

$F(y) \sim \frac{a_0}{2} + \sum a_k \cos ky + b_k \sin ky$ is

a standard Fourier series, so

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} F(y) \cos ky \, dy$$

$y = \frac{\pi x}{\ell}$

$dy = \frac{\pi}{\ell} dx$

$$= \frac{1}{\ell}\int_{-\ell}^{\ell} f(x) \cos\left(\frac{k\pi x}{\ell}\right) dx$$

so $f(x) \sim \frac{a_0}{2} + \sum a_k \cos\left(\frac{\pi k x}{\ell}\right) + b_k \sin\left(\frac{\pi k x}{\ell}\right)$

$$a_k = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x) \cos\left(\frac{\pi k x}{\ell}\right) dx$$

$$b_k = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x) \sin\left(\frac{\pi k x}{\ell}\right) dx.$$

If f is even, all the b_k disappear

and by symmetry, $f(x)\cos(\frac{k\pi x}{l})$ is even,

So $$a_k = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{\pi k x}{l}\right)dx.$$

If f is odd, all the a_k disappear,

$f(x)\sin(\frac{k\pi x}{l})$ is even,

and $$b_k = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{\pi k x}{l}\right)dx.$$