Name: _____

Math 418 (Tully-Doyle) **Exam 2** **April 21, 2026**

Note: You may leave arithmetic and trigonometric quantities unsimplified. **No notes. No calculators.** *Show all your work and clearly indicate your answer for full credit. Partial solutions will receive partial credit.*

Question	Points	Score
1	6	
2	10	
3	8	
4	16	
Total:	40	

a

Initials:

1. (6 points) Use separation of variables to find the forms of the possible separated solutions $u(x,t) = X(x)T(t)$ to the differential equation $u_{tt} + 4u_{xx} = 0$. (Just identify the structures of the solutions based on λ, don't find eigenvalues!)

Initials:

a

Initials:

2. (10 points) The differential equation $u_t = ku_{xx}$ with $u(-1,t) = u(1,t)$ and $u_x(-1,t) = u_x(1,t)$leads to the system of equations

$$T' + k\lambda T = 0$$
$$X'' + \lambda X = 0,$$

and the boundary conditions $X(-1) = X(1)$ and $X'(-1) = X'(1)$. Find the eigenvalues and corresponding eigenfunctions of the system corresponding to positive values of λ. Write a general solution $u(x,t) = \sum T_n(t)X_n(x)$.

Initials:

a

Initials:

3. (8 points) Given the system of equations

$$T' + k\lambda T = 0$$
$$X'' + \lambda X = 0,$$

and the boundary conditions $X(-1) = X(1)$ and $X'(-1) = X'(1)$, determine if it is possible for λ to be an eigenvalue if $\lambda < 0$ or $\lambda = 0$. If so, find the eigenfunctions corresponding to λ.

Initials:

a

Initials:

4. (16 points) Consider the function $f(x) = 1 - x^2$ on $(-1, 1)$.
   (a) Show that $f(x) \perp \sin(\pi n x)$ for $n = 1, \ldots$.
   (b) Find a full Fourier series representation for the function $f(x)$.

Initials:

a

Problem 4 cont.

(c) Show that the Fourier series for f converges pointwise to f on $[-1, 1]$.

(d) Use your result to show that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -\frac{\pi^2}{12}$. (Hint: plug in a carefully chosen value of x.)

Initials:

a

Initials:

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Initials:

a

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