Name: _____

Math 418 (Tully-Doyle) **Exam 1** **April 21, 2026**

No notes. No calculators. *Show all your work and clearly indicate your answer for full credit. Partial solutions may receive partial credit.*

Question	Points	Score
1	8	
2	8	
3	9	
4	10	
5	10	
Total:	45	

1. (8 points) For each equation, classify it as linear and homogeneous, linear and inhomogeneous, or non-linear.

   (a) (a) $u_{xt} + 2u_{tt} + u_t = \sin x + t$

   (b) $t^2(u_{xx} - u_{tt}) = 0$

   (c) $u_t + ku_{xt} + uu_t = 0.$

   (d) Show that $u = \cos(x)\sin(10t)$ is a solution to the wave equation $u_{tt} = 100u_{xx}$. What initial conditions does it satisfy?

2. (8 points) Find the solution $u = u(x, y)$ to the PDE

$$u_x - 3x^2 y\, u_y = 0$$
$$u(0, y) = 2y.$$

3. (9 points) (a) Find the characteristic change of variables for the equation

$$u_x + 4u_y - 2u = 2.$$

(b) Use the change of variables from part (a) to solve the PDE.

4. (10 points) (a) Find the general solution of the PDE

$$u_{xx} + u_{xt} - 6u_{tt} = 0.$$

(b) The PDE given by $u_{tt} - 4u_{xt} + 3u_{xx} = 0$ has general solution $u(x,t) = f(x + \frac{1}{3}t) + g(x + t)$ for arbitrary C^2 functions f, g. Use this to solve the initial value problem

$$\begin{aligned} u_{xx} - 4u_{xt} + 3u_{tt} &= 0, \\ u(x,0) &= x, \\ u_t(x,0) &= e^x. \end{aligned}$$

5. (10 points) Solve the heat equation with variable dissipation

$$\begin{aligned} u_t - ku_{xx} + e^{-2t}u &= 0, \;\; x \in (-\infty, \infty), \; t > 0 \\ u(x,0) &= \phi(x) \end{aligned}$$

by letting $u = (e^{\frac{1}{2}e^{-2t}})v$.