6.1: 31–46

6.2: 1–16
37...

6.3: 1–15

6.1 Intro to determinants.

$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$

$ad - bc = 0$ means that $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

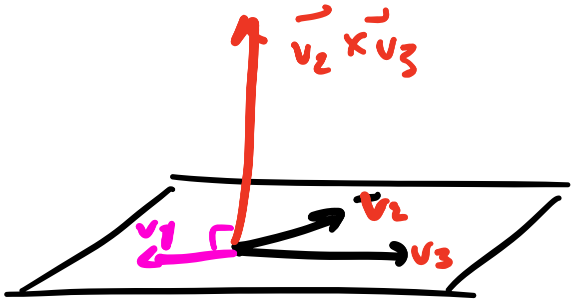
makes no sense. so

$ad - bc = 0 \Rightarrow A$ is not invertible.

How can we do this for 3×3 matrices?

$A = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 \end{bmatrix}.$

we want a number that will tell us if there is a redundant vector in $\vec{v}_1, \vec{v}_2, \vec{v}_3$, which would imply that $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are in the same plane.



if $(\vec v_1 \times \vec v_2) \perp \vec v_3$, then $\vec v_3$ is in the same plane with $\vec v_1$ and $\vec v_2$.

So if $\vec v_1 \cdot (\vec v_2 \times \vec v_3) = 0$,

$\vec v_1, \vec v_2, \vec v_3$ are in the same plane and hence not linearly independent.

Def: If $A = [\vec v_1 \ \vec v_2 \ \vec v_3]$ is 3×3,

$$\det A = \vec v_1 \cdot (\vec v_2 \times \vec v_3).$$

3×3 A is invertible iff $\det A \neq 0$.

---

quick calc

$$\begin{array}{ccccc} & & - & - & - \\ \left[\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}\right] & & & \begin{matrix} a_{11} & a_{12} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \end{matrix} & \\ & & + & + & + \end{array}$$

Ex: $\det \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{matrix}$$

$$(1)(1)(-1) + 0(1)(1) + 3(0)(2)$$

$$- (1)(1)(3) - (2)(1)(1) - (-1)(0)(0)$$

$$= -1 + 0 + 0 - 3 - 2 - 0$$

$$\boxed{= -6}$$

$\vdots$

Important example:

Find values for which

$$A = \begin{bmatrix} \lambda & 1 & 1 \\ 1 & \lambda & -1 \\ 1 & 1 & \lambda \end{bmatrix} \text{ is not invertible}$$

$$\det A = \lambda^3 - 1 + 1 - \lambda + \lambda - \lambda$$

$$= \lambda^3 - \lambda = \lambda(\lambda-1)(\lambda+1)$$

So $\det A = 0$ if $\lambda = 0, 1, -1$.