Trig Integrals - 7.2

which identities do we need?

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

pythagorean identities.

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\cos^2 x = \frac{1 + \cos 2x}{2}$

half-angle identities

Lets look at some examples:

$\int \sin^2 x \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$= \int u^2 \, du$

$= \frac{1}{3} u^3 + C$

$= \frac{1}{3} \sin^3 x + C.$

when do we know we can use a u-sub in a sine/cosine integral? need an extra function to be the du.

$$\int \sin^3 x \cos^2 x \, dx$$

sin has odd power.

$$= \int \sin^2 x \cos^2 x \, \underbrace{(\sin x \, dx)}$$

reserved to be du.

u should be cos x

these aren't cosines.

$$= \int (1-\cos^2 x)\cos^2 x \, (\sin x \, dx)$$

$u = \cos x$

$du = -\sin x \, dx$

$$= \int (1-u^2) u^2 (-du)$$

$$= -\int u^2 - u^4 \, du$$

$$= -\frac{1}{3}u^3 + \frac{1}{5}u^5 + C = -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$$

Lesson: $\int \cos^m x \sin^n x$.

If m or n is odd,

reserve one sine or cosine to be du.
convert the rest into the other function.

u-sub

$$\int \cos^5 x \, \underline{\sin^5} x \, dx$$

$$\int \cos^4 x \sin^5 x \, (\cos x \, dx) \quad \text{since}$$

$$= \int (\cos^2 x)^2 \sin^5 x \, (\cos x \, dx)$$

$$= \int (1-\sin^2 x)^2 \sin^5 x \, (\cos x \, dx) \quad \text{convert}$$

$$= \int (1-u^2)^2 u^5 \, du \qquad \text{u-sub.}$$

$$\int \cos^2 x \, dx \qquad \text{not odd.}$$

half-angle!

$$= \int \frac{1}{2} + \frac{1}{2}\cos 2x \, dx$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

$$\int \cos^2 x \sin^2 x \, dx$$

no odd power.
half angle.

$$= \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$$

$$= \int \frac{1}{4} - \frac{1}{4}\cos^2 2x \, dx$$

another even power! half-angle.

$$= \int \frac{1}{4} - \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos 4x\right) dx$$

$$= \int \frac{1}{4} - \frac{1}{8} - \frac{1}{8}\cos 4x \, dx$$

$$= \int \frac{1}{8} - \frac{1}{8}\cos 4x \, dx$$

$$= \frac{1}{8}x - \frac{1}{32}\sin 4x + C$$

lesson: $\int \cos^m x \sin^n x \, dx$

If m, n even, reduce using half-angle identities other cases:

$u = \tan x \quad du = \sec^2 x\,dx$

even power of $\sec x$.

reserve $\sec^2 x$,

convert the rest to tangent

$\sec^2 x$ to $\tan^2 x + 1$

$u = \sec x \quad du = \tan x \sec x\,dx$

odd power of tangent

reserve $\tan x \sec x$

convert $\tan^2 x$ to $\sec^2 x - 1$.

$\int \tan^3 x \sec^2 x\,dx$ odd tangent.

$= \int \tan^2 x \sec x\,(\tan x \sec x\,dx)$ reserve

$= \int (\sec^2 x - 1) \sec x\,(\tan x \sec x\,dx)$ convert

$= \int (u^2 - 1)u\,du$

u-sub.

$u = \sec x$

$du = \sec x \tan x\,dx$.

Useful: $\int \tan x\,dx = \ln|\sec x| + C$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C.$

Sometimes we just need to be clever.

$$\int \tan^3 x\,dx = \int \tan^2 x \tan x\,dx$$

$$= \int (\sec^2 x - 1) \tan x\,dx$$

$$= \int \sec^2 x \tan x\,dx - \int \tan x\,dx$$

$$= \frac{1}{2}\tan^2 x - \ln|\sec x| + C.$$