7.1 Integration by Parts

We're going to start expanding what we can integrate. Integration can be thought of as "undoing" derivatives.

Table Integrals $\longleftrightarrow$ **Table Derivatives**

$$\frac{d}{dx}\sin x = \cos x \quad \longleftrightarrow \quad \sin x + C = \int \cos x \, dx$$

Chain Rule $\longleftrightarrow$ **U-Substitution**

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x) \quad \longleftrightarrow \quad \int f'(g(x))\, g'(x)\, dx$$

$$= \int f'(u)\, du$$

$$= f(g(x)) + C.$$

Now we'll consider how to undo the product rule.

$$\frac{d}{dx}(fg) = f'g + fg'$$

$$\text{so } fg = \int f'g + fg' \, dx$$

$$fg = \int f'g\,dx + \int fg'\,dx$$

$$fg - \int f'g\,dx = \int fg'\,dx$$

$u = f \qquad v = g$

$du = f'\,dx \qquad dv = g'\,dx$

$$uv - \int v\,du = \int u\,dv$$

This is called integration by part.

Ex: $\int \boxed{x}\,\boxed{e^x dx}$

$\quad\; u \quad dv$

$u = x$ ↓ diff $du = dx$

$dv = e^x dx$ ↓ int $v = e^x$

$$= uv - \int v\,du$$

$$= xe^x - \int e^x dx$$

$$\boxed{= xe^x - e^x + C}$$

Ex: $\int x^2 e^x dx$

$u = x^2 \qquad dv = e^x dx$

$du = 2x\,dx \qquad v = e^x$

$= x^2 e^x - \underbrace{\int 2xe^x dx}_{\text{again!}}$

$u = 2x \qquad dv = e^x dx$

$du = 2dx \qquad v = e^x$

$= x^2 e^x - \left(2xe^x - \int 2e^x dx\right)$

$= x^2 e^x - 2xe^x + 2e^x + C$

How do we choose u?

- Logarithm
- Inverse Trig
- Polynomial
- Exponential
- Trigonometric

$$\int \ln(x)\,dx$$

Logarithm. (nothing else to try)

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$$= x\ln(x) - \int x\left(\frac{1}{x}\right)dx$$

$$= x\ln(x) - \int dx$$

$$\boxed{= x\ln(x) - x + C}$$

$$\int e^x \sin x\,dx$$

$u = e^x \qquad dv = \sin x\,dx$

$du = e^x dx \qquad v = -\cos x$

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$$= -e^x\cos x - \int(-\cos x)e^x dx$$

$$= -e^x\cos x + \int e^x \cos x\,dx$$

$u = e^x \qquad dv = \cos x\,dx$

$du = e^x dx \qquad v = \sin x$

$$= -e^x\cos x + \left(e^x\sin x - \int e^x \sin x\,dx\right)$$

$$= -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx$$

$$\underbrace{\int e^x \sin x\,dx}_{I} = -e^x\cos x + e^x\sin x - \underbrace{\int e^x\sin x\,dx}_{I}$$

$$2\int e^x \sin x\,dx = -e^x\cos x + e^x \sin x$$

$$\int e^x \sin x\,dx = -\frac{1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + C$$