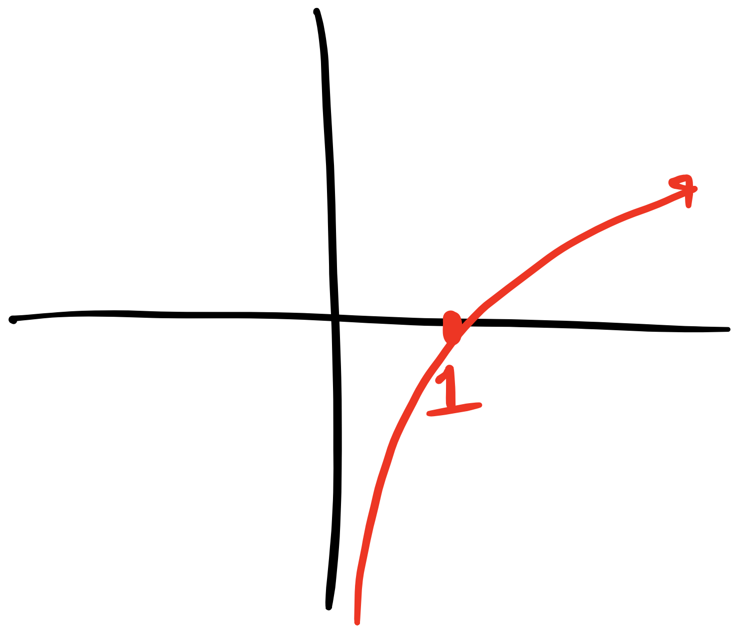
# 6.3 Natural Exponential.

Lets draw $y = \ln(x)$.

$\lim_{x \to \infty} \ln(x) = \infty$

$\lim_{x \to 0^+} \ln(x) = -\infty$

$\ln(1) = 0$




Notice that $\ln(x)$ must have an inverse function, since it is one-to-one on $(0,\infty)$

Lets draw it!

first, we call it $\exp(x) = y$ iff $x = \ln(y)$

$y = \ln(x)$ so $y = \exp(x)$

D: $(0,\infty)$ D: $(-\infty,\infty)$

R: $(-\infty,\infty)$ R: $(0,\infty)$

$(1,0)$ $(0,1)$

since $\ln(1) = 0$ so $\exp(0) = 1$.

one other special number:

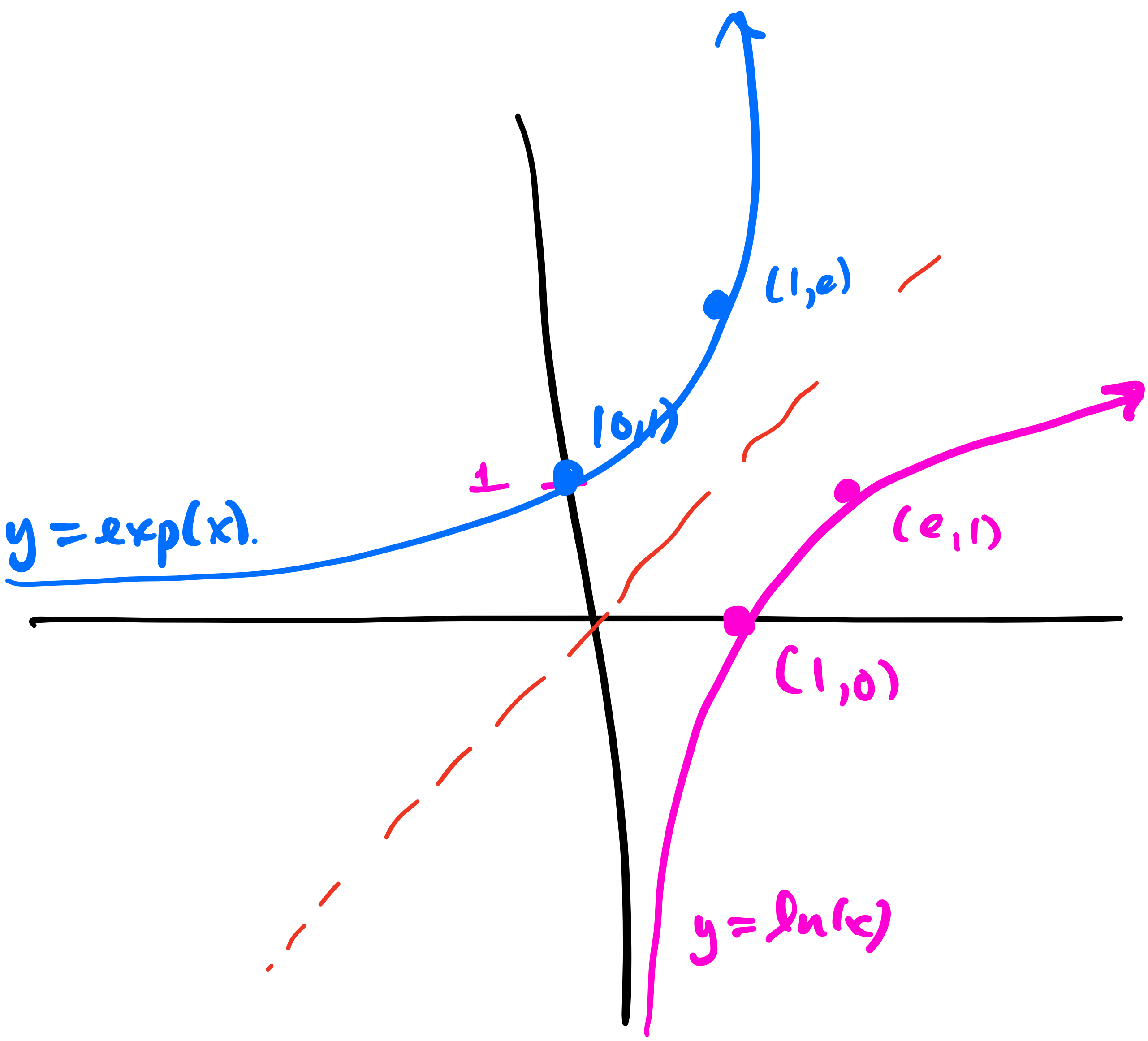
$\ln x$ has to pass 1 somewhere.

(it is increasing, and is below 1 and above 1)

<u>**Def:**</u> $e$ is the <u>unique number</u> so that

$\ln(e) = 1.$

of course this also means that $\exp(1) = e.$



In fact, since $\ln(e^r) = r\ln(e) = r$,

$$\exp(r) = e^r.$$

So $\exp(x) = e^x$ is the inverse function for $\ln(x)$.

$$e^{\ln(x)} = x \quad \text{for } x>0$$

$$\ln(e^x) = x \quad \text{for all } x.$$

These "undo" each other.

<u>Ex:</u> find $x$ if $\ln x = 5$

$$e^{(\ln x)} = e^{(5)}$$

$$x = e^5.$$

<u>Ex:</u> solve $e^{5-3x} = 10$

$$\ln\left(e^{5-3x}\right) = \ln(10)$$

$$5-3x = \ln(10)$$

$$-3x = \ln(10)-5$$

$$x = \frac{\ln(10)-5}{-3}$$

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limits: $\lim_{x\to-\infty} e^x = 0$

$$\lim_{x\to\infty} e^x = \infty.$$

properties:

① $e^{x+y} = e^x e^y$

② $e^{x-y} = \frac{e^x}{e^y}$

③ $(e^x)^y = e^{xy}$

why? $\ln(e^x e^y) = \ln(e^x) + \ln(e^y)$

$= x + y.$

so $e^{\ln(e^x e^y)} = e^{x+y}$

$e^x e^y = e^{x+y}.$

(2)(3) left for you.

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Property: $\frac{d}{dx} e^x = e^x$ !

(in fact, there is exactly <u>one</u> function with this property)

<u>Ex</u>: $\frac{d}{dx} e^{\tan x} = e^{\tan x} \cdot \frac{d}{dx}(\tan x)$

$= e^{\tan x} \sec^2 x.$

<u>So</u> $\int e^x \, dx = e^x + C.$ easy peasy.

Ex: $\int x^2 e^{x^3} dx$

let $u = x^3$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$= \int e^u \left(\frac{1}{3} du\right)$

$= \frac{1}{3} e^u + C$

$= \frac{1}{3} e^{x^3} + C$