Time to use our new toolset.

Recall: Newton's second law:

$$F = ma = m\frac{d^2s}{dt^2}$$

Work:

$\underbrace{W = Fd}$ where F is force and d is displacement.

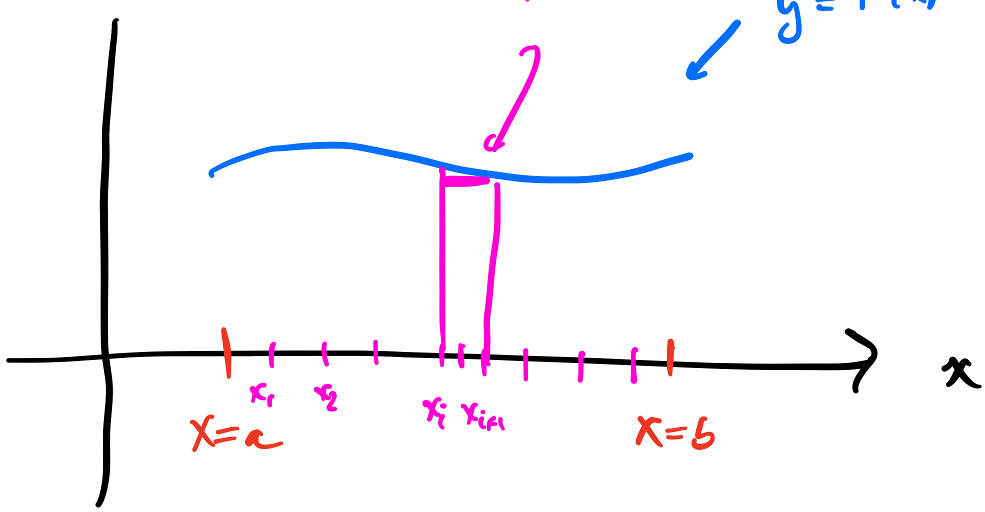
high school work.

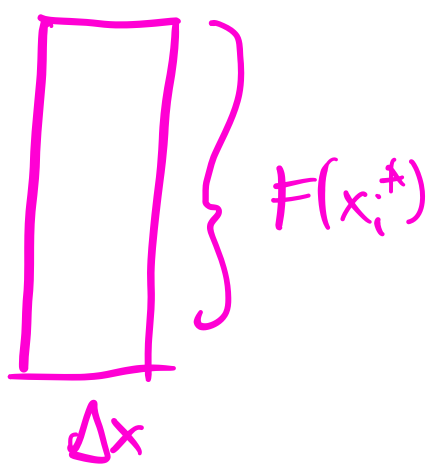
what if F isn't constant?

$F(x)$ = force

force is almost constant

$y = F(x)$ = force





$$W_{tiny} = \underbrace{F(x_i^*)}_{F}\underbrace{\Delta x}_{d}$$

$$W \approx \sum_{i=1}^{n} F(x_i^*)\Delta x$$

$$W_{Total} = \int_a^b F(x)\,dx$$

Big work is the integral of small work

Example:

mass attached to spring.

$x = 0$ = equilibrium position

Spring law: Force required to displace mass is proportional to displacement.

$$F(x) = kx$$

k is called the <u>spring</u> <u>constant</u>.

A force of 40N is required to hold a spring that has been stretched from a natural length of 10 cm to a length of 15cm. How much work is done stretching from 15cm to 18cm?

Concern 1: units. $N = kg \cdot m / s^2$

10 cm = .1 m

15cm = .15 m

18cm. = .18m

equilibrium] displacement : .05

Concern 2: what is k?

$F(x) = kx$

$F(.05) = 40N$

$k(.05) = 40$

$k = 800$

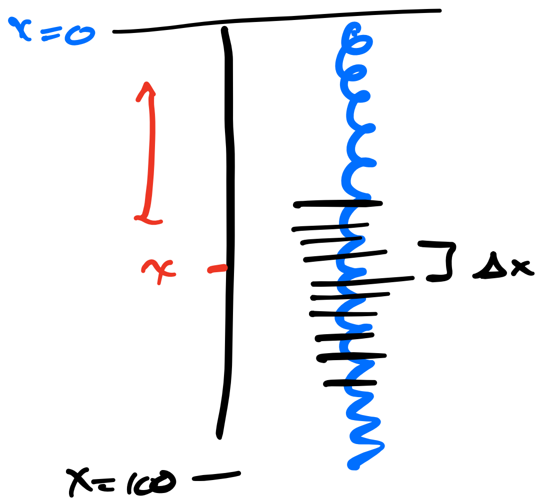
so $F(x) = 800x$

Concern 3: what is W .05 → .08?

$$W = \int_{.05}^{.08} F(x)\,dx = \int_{.05}^{.08} 800x\,dx = 400x^2 \Big|_{.05}^{.08}$$

$$\boxed{= 1.56J}.$$

Example: 200 lb cable is 100 ft long and hangs from the top of a building. how much work is required to pull it up?



each chain chunk is Δx long.
the density of the cable is $200\text{lb}/100\text{ft} = 2\ \text{lb}/\text{ft}$

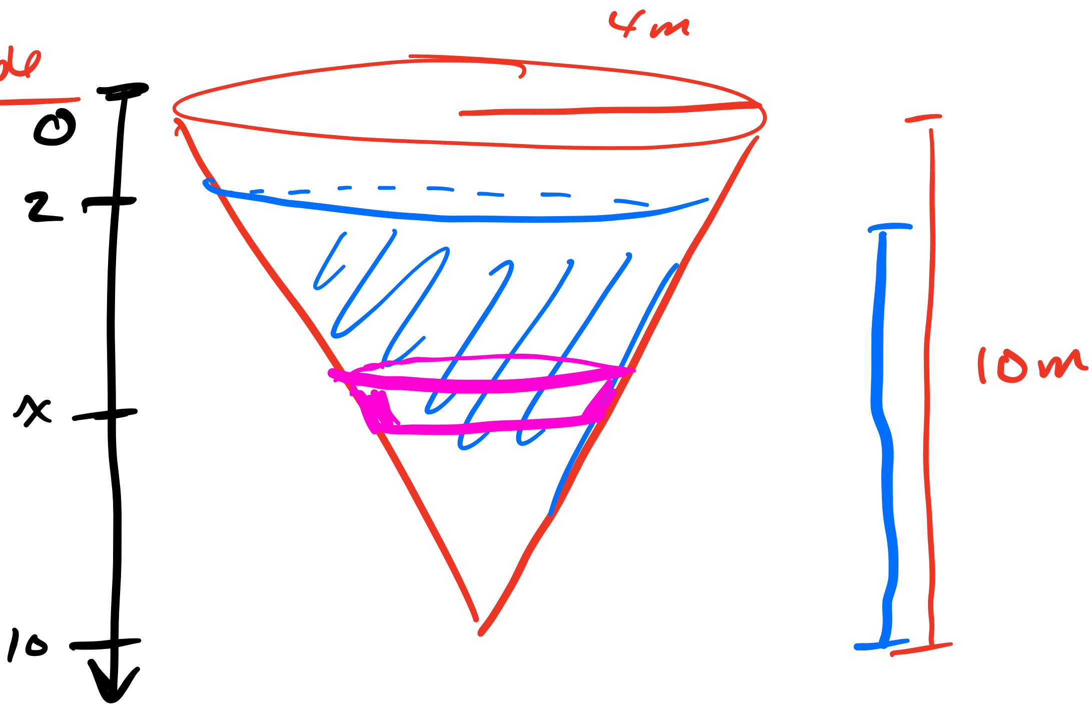
so the weight of the chunk is $2\Delta x$. lb (which is already a force).

how far to lift it? x ft.

$$\text{small work} = (2\Delta x)(x)$$
$$= 2x\Delta x \quad \text{lb-ft.}$$

$$\text{big work} = \int_0^{100} 2x\,dx.$$
$$= 10000 \text{ lb-ft}$$

Example



how much work to pump the water out of the top of the tank?

$\rho = 1000\ kg/m^3$.

small work: get the slab out of the tank.

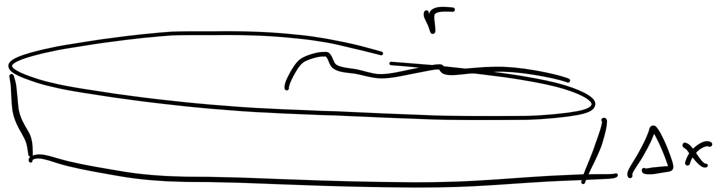
: $W = F\underline{d}$ lift the slab x m.

$F = m \cdot a = mg. = m(9.8)$

- so now we need to find the mass of the slab.

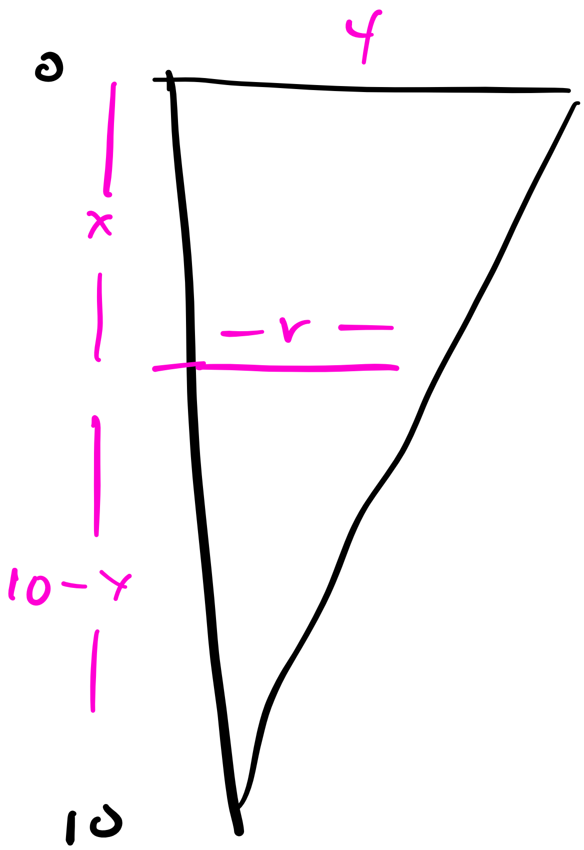
mass = volume · density = volume $(1000\ kg/m^3)$.

volume:



$$V = \pi r^2 \Delta x$$

the whole question comes down to finding what r is at height x



$$\frac{4}{10} = \frac{r}{10-x} \qquad r = \frac{4}{10}(10-x)$$

$$V = \pi\left(\frac{2}{5}(10-x)\right)^2 = \frac{4}{25}\pi(10-x)^2\Delta x$$

$$m = V\rho = \frac{4}{25}\pi(10-x)^2(1000) = 160\pi(10-x)^2\Delta x$$

$$F = mg = 160\pi(10-x)^2(9.8)\Delta x$$

$$= 1568\pi(10-x)^2\Delta x$$

$$W_{small} = Fd = 1568\pi(10-x)^2 x\,\Delta x$$

$$W_{big} = \int_2^{10} W_{small} = \int_2^{10} 1568\pi\, x(10-x)^2\,dx$$

$$\boxed{\doteq 3.4\times10^6\,J.}$$