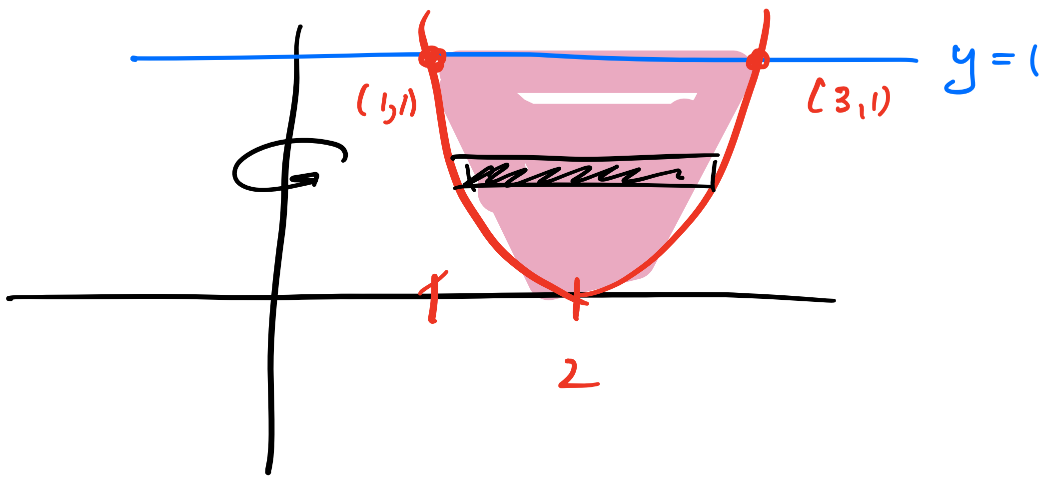
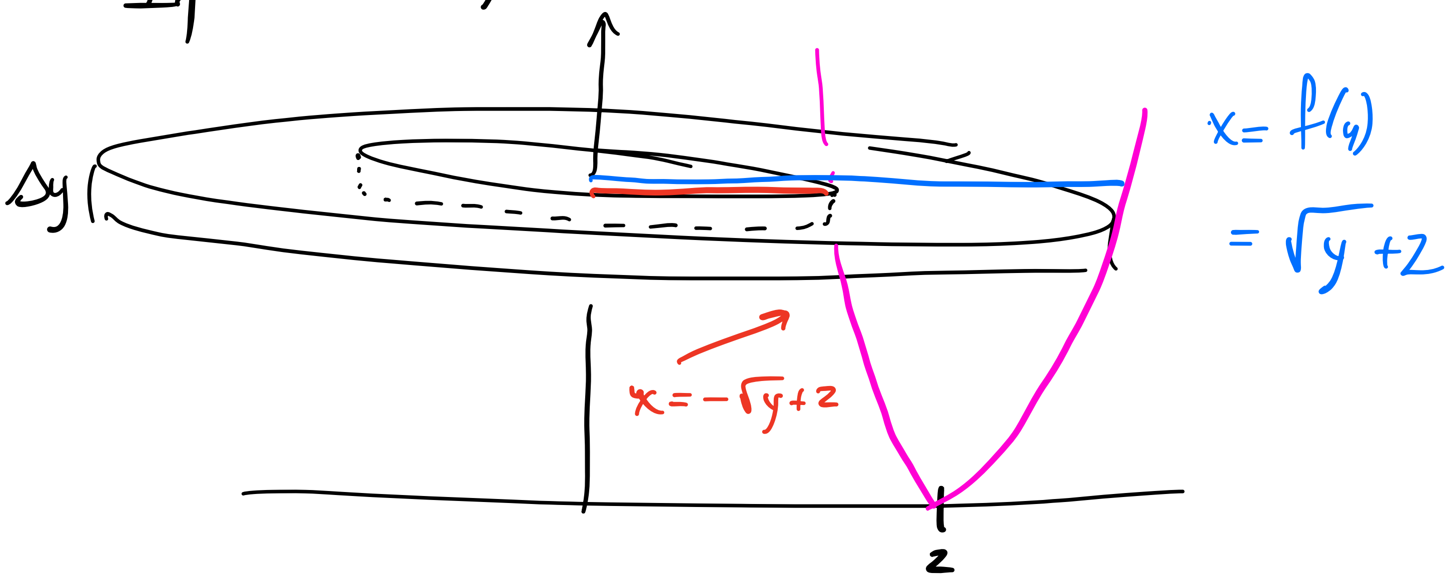
Consider the region bounded by
$y = (x-2)^2$ and $y = 1$;
rotated about the y-axis.



If we try to use washers, we get



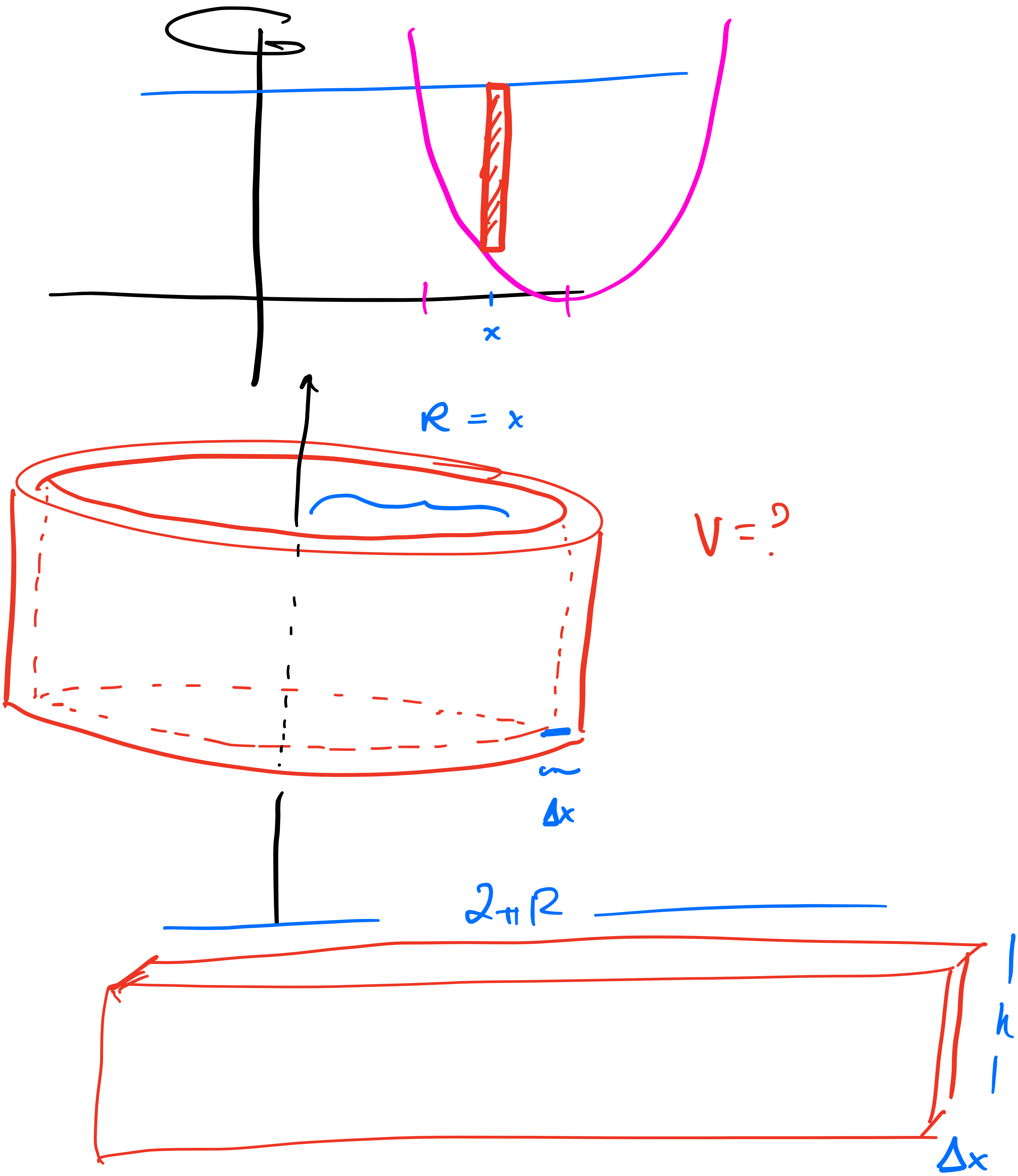
yuck!

$$\int_0^1 \pi\left(\left(\sqrt{y}+2\right)^2 + \left(-\sqrt{y}+2\right)^2\right) dy$$

this works but it's ugly!
maybe there's a more natural way to

do this

maybe we can cut the other way.



$$V_{shell} = 2\pi R h \Delta x$$

$= 2\pi x\left(1-(x-2)\right)^2 \Delta x.$

$$V = \int_1^3 V_{shell} = \int_1^3 2\pi x\left(1-(x-2)^2\right) dx$$

$$= \int_1^3 2\pi x\left(1-(x^2-4x+4)\right) dx$$

$$= \int_1^3 2\pi x\left(-x^2+4x-3\right) dx$$

$$= 2\pi \int_1^3 -x^3+4x^2-3x \; dx$$

$$= 2\pi\left(\frac{-x^4}{4}+\frac{4}{3}x^3-\frac{3}{2}x^2\right)\Big|_1^3$$

$$= 2\pi\Bigg(\left(\frac{-81}{4}+\frac{4\cdot 27}{3}-\frac{3}{2}\cdot 9\right) - \left(-\frac{1}{4}+\frac{4}{3}-\frac{3}{2}\right)\Bigg) -$$

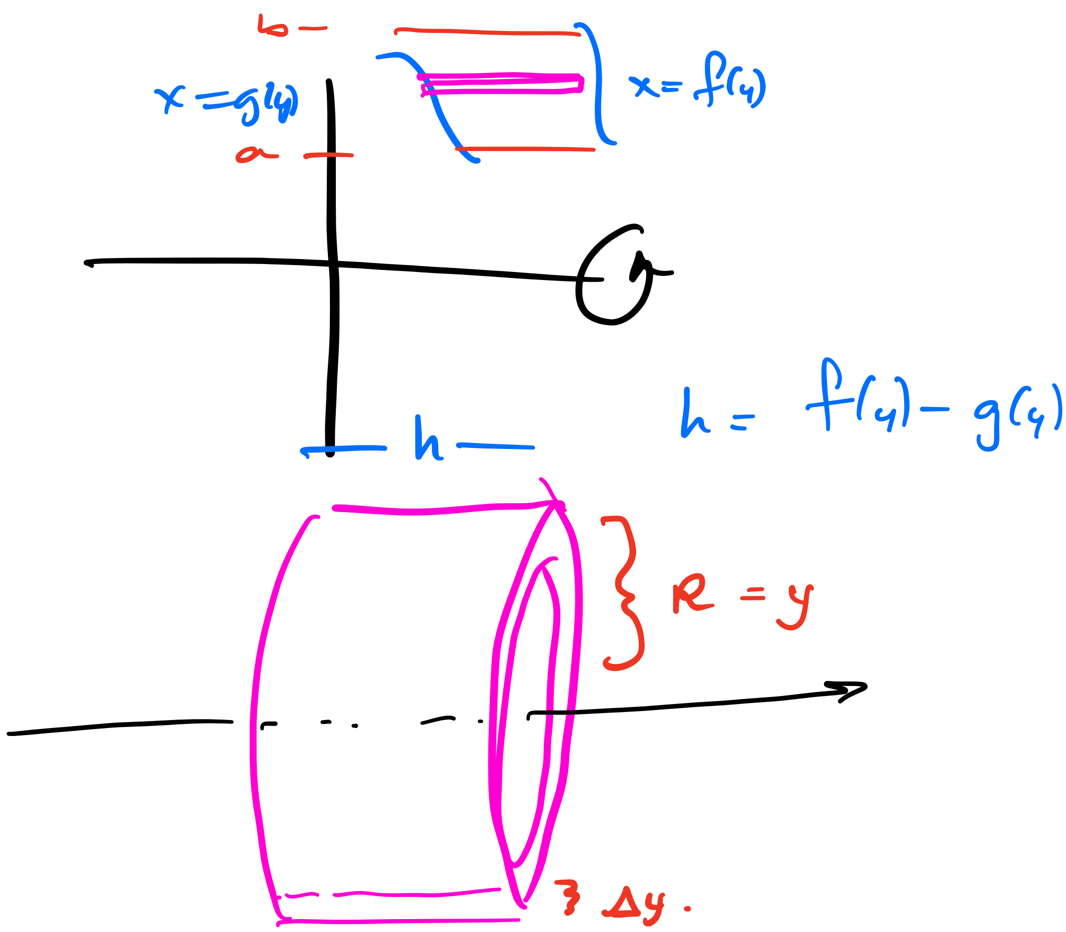
General formula for shells

$$V = \int_a^b 2\pi R\, h\, dx$$

or

$$\int_a^b 2\pi R\, h\, dy$$

General Picture



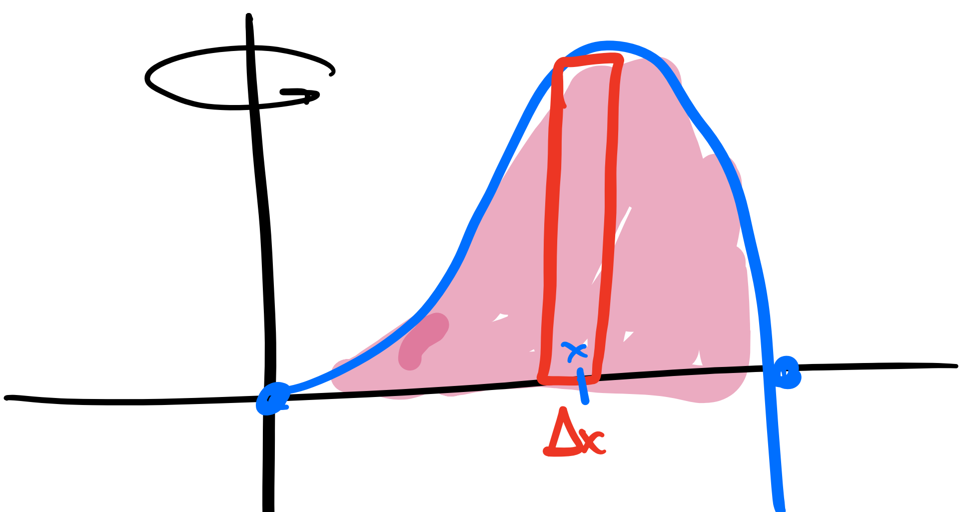
$$V_{slice} = 2\pi R\, h\, \Delta y$$

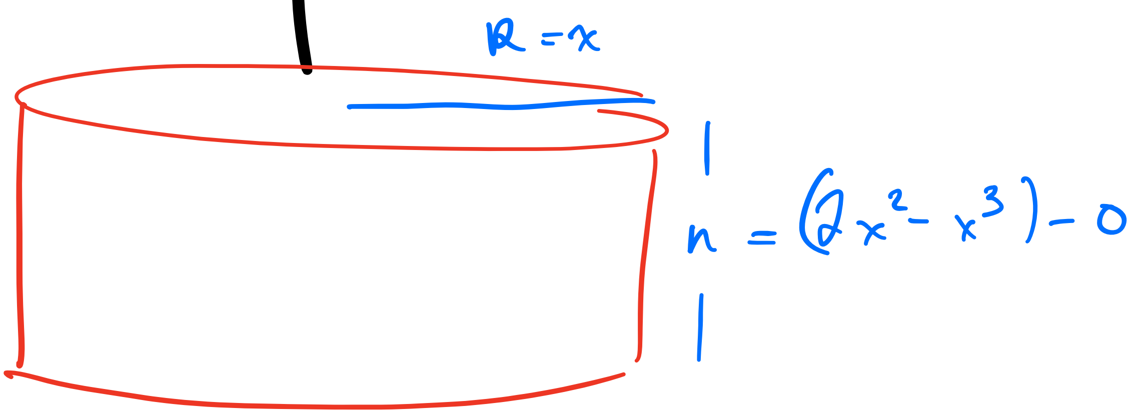
$$= 2\pi y \left(f(y) - g(y) \right) \Delta y$$

$$V = \int_a^b 2\pi y \left(f(y) - g(y) \right) dy.$$

Example:

Find the volume of the solid obtained by rotating the region contained between the x-axis and $y = 2x^2 - x^3$ about the y-axis.





$$V_{shell} = 2\pi x(2x^2 - x^3)\Delta x$$

$$V = \int_0^2 2\pi x(2x^2 - x^3)\,dx$$

<u>Example</u> (class to do)

Compute the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $x = 4$, $x = 0$, $y = 0$ about the x-axis. Do it with both shells and disks.