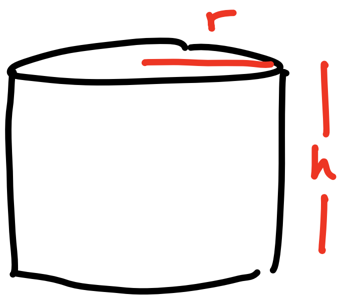
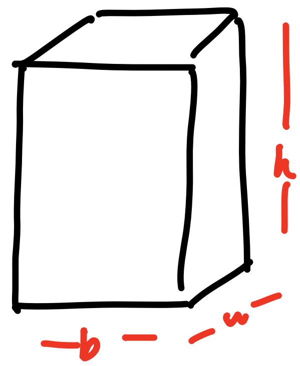
The basic idea of volume is to use the simple formula for the volume of a cylinder.



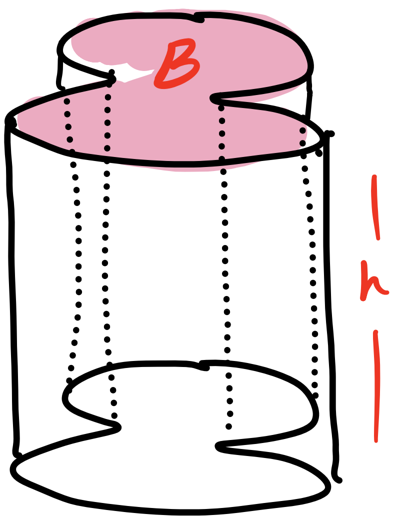
$$B = \pi r^2$$

$$V = Bh = \pi r^2 h$$



$$B = bw$$

$$V = Bh = bwh.$$



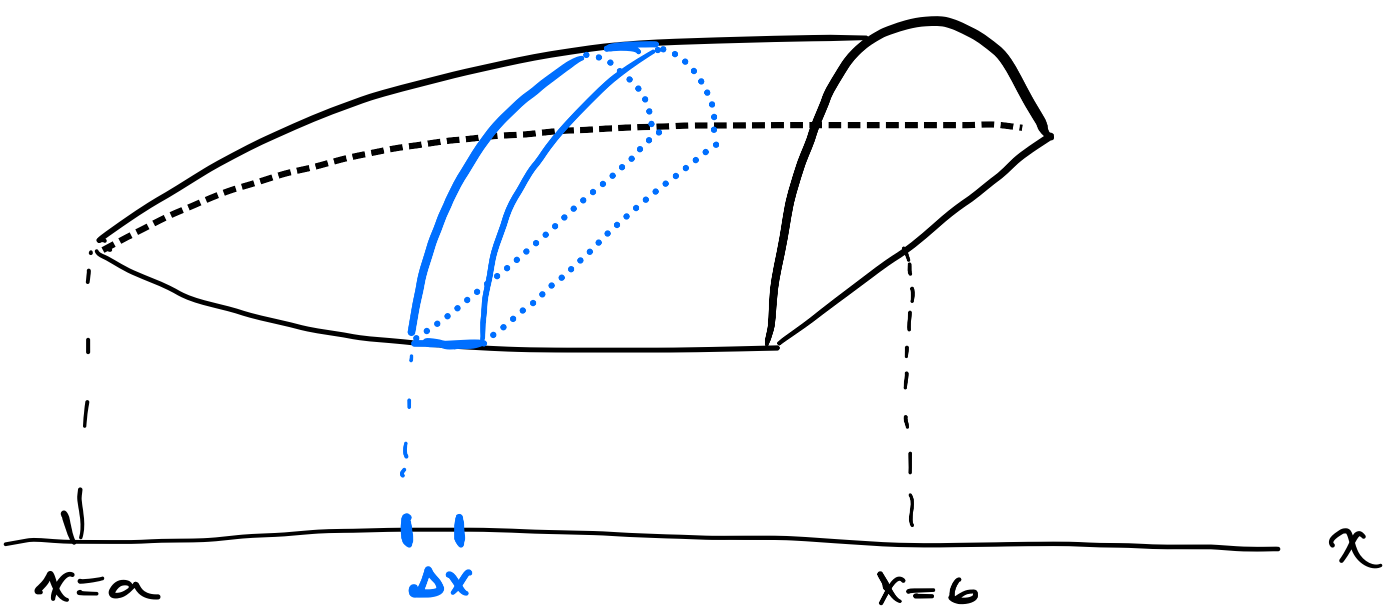
$$V = Bh$$

Basic approach to volume

1. decompose a solid into thin cross-sections

2. Compute the volume of cross-section using $V = B\Delta h$.

3. add up the volumes with an integral



typical slice:

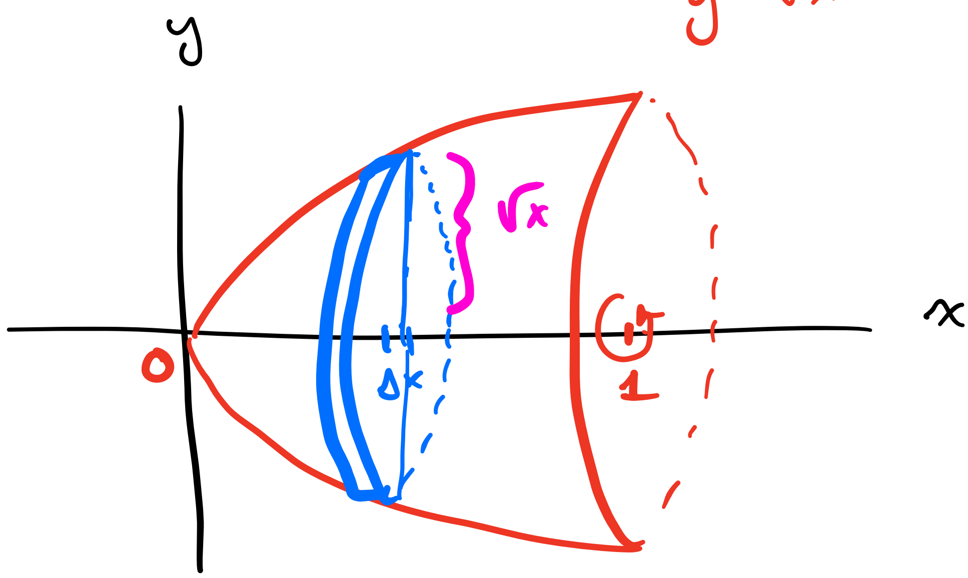
$V_{slice} = B\Delta x$

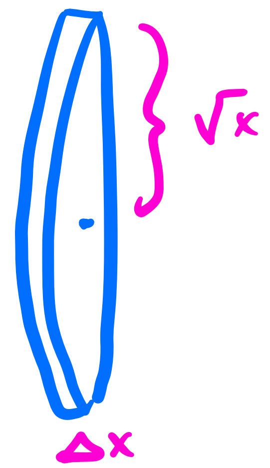
volume of the solid

$$V = \int_a^b B\,dx.$$

<u>important</u>

Draw a picture!





$B =$ circle area

$= \pi(\sqrt{x})^2$

$V_{slice} = B\Delta x$

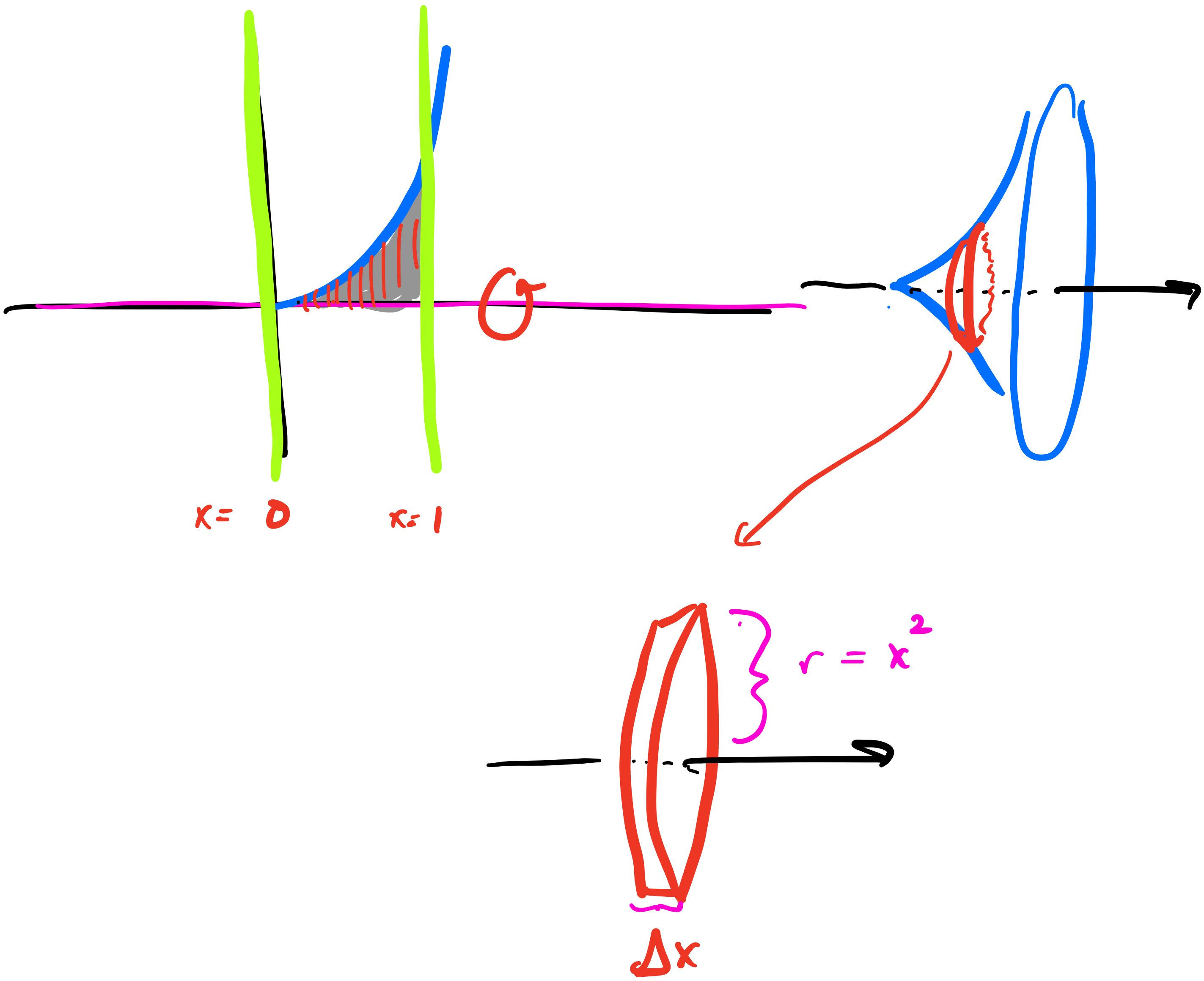
$= \pi(\sqrt{x})^2\Delta x$

$= \pi x \Delta x.$

location of slices

$$\text{Volume} = \int_0^{1} \underbrace{\pi x\,dx}_{\text{volume of typical slices}}$$

Example: volume of surface obtained by rotating region bounded by x-axis, $y = x^2$, $x=0$, $x=1$ about the x-axis

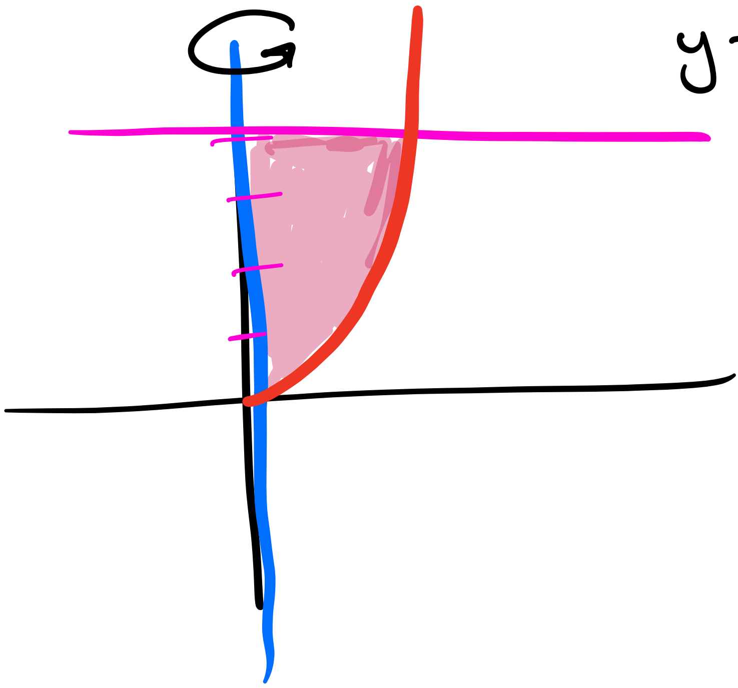


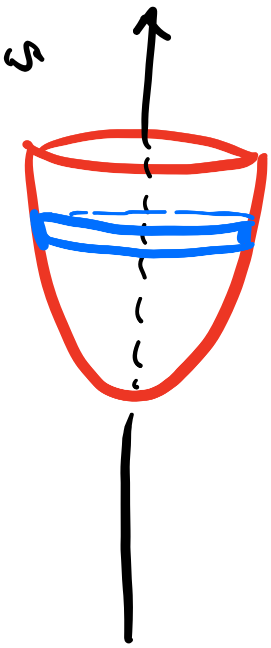
$$V_{slice} = \pi r^2 h$$
$$= \pi (x^2)^2 \Delta x$$
$$= \pi x^4 \Delta x$$

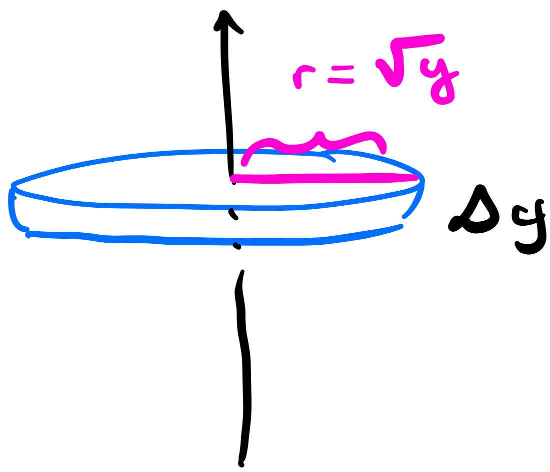
$$V = \int_{x=0}^{x=1} V_{slice} = \int_0^1 \pi x^4 \, dx = \frac{1}{5} \pi x^5 \Big|_0^1$$
$$= \frac{1}{5} \pi .$$

Example 2:

region bounded by $\underline{y\text{-axis}}$, $\underline{y = 4}$, $\underline{\underline{y = x^2}}$, rotated about y-axis







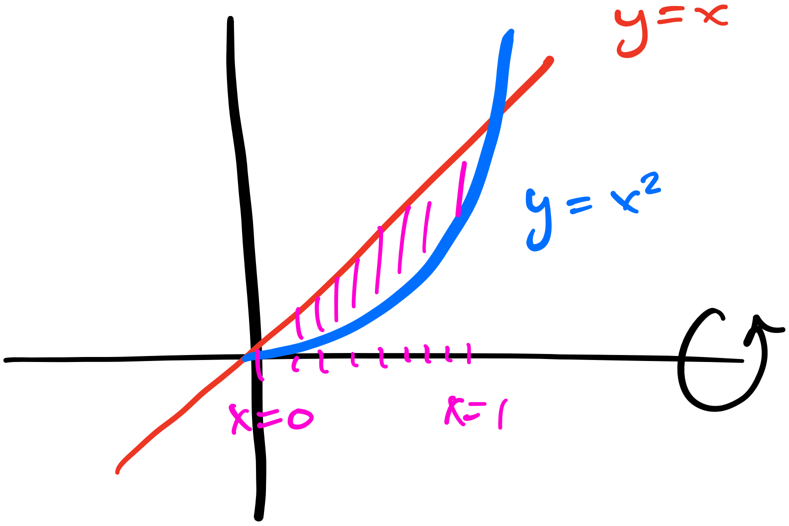
$$V_{slice} = \pi r^2 h = \pi(\sqrt{y})^2 \Delta y$$

$$= \pi y \, dy$$

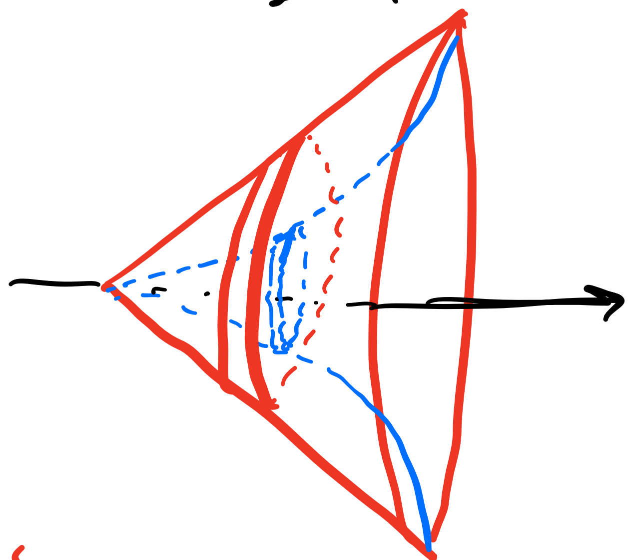
$$V = \int_0^4 \pi y \, dy = \frac{\pi}{2} y^2 \Big|_0^4 = 8\pi.$$

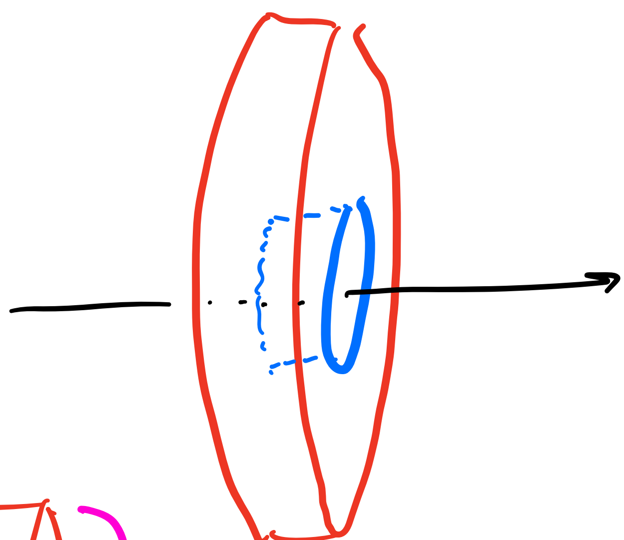
Example 3 (washers)

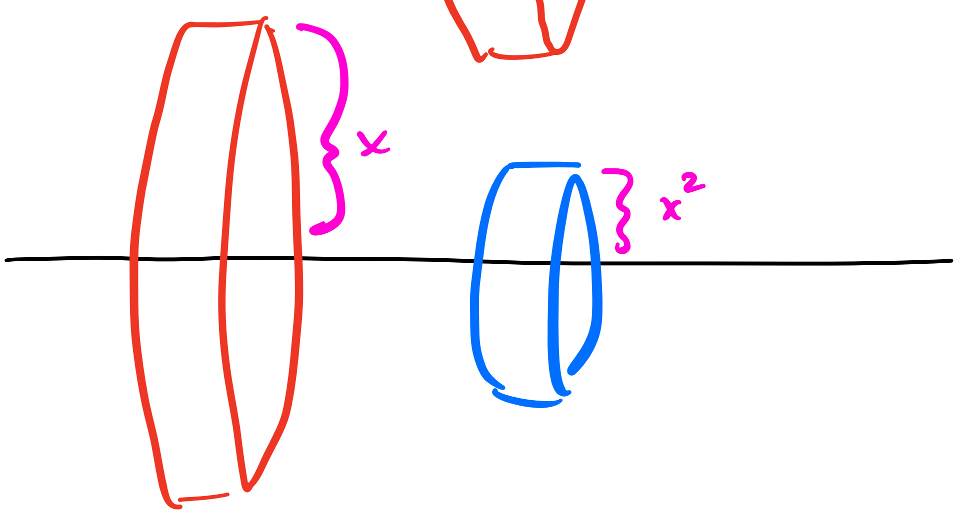
rotate the region bounded by $y=x^2$ and $y=x$ about the x-axis. Find the volume of the resulting solid.



Sort of like the bell of a horn.







$$V_{slice} = V_{BIG\ slice} - V_{small\ slice}$$

$$= \pi (x)^2 \Delta x - \pi (x^2)^2 \Delta x$$

$$= \pi (x^2 - x^4) \Delta x$$

$$V = \int_0^1 V_{slice} = \int_0^1 \pi (x^2 - x^4)\,dx$$

$$= \pi \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{5} \right)$$

General formulas:

disks about

x-axis : $\int_a^b \pi (f(x))^2\,dx$

y-axis : $\int_a^b \pi (g(y))^2\,dy$

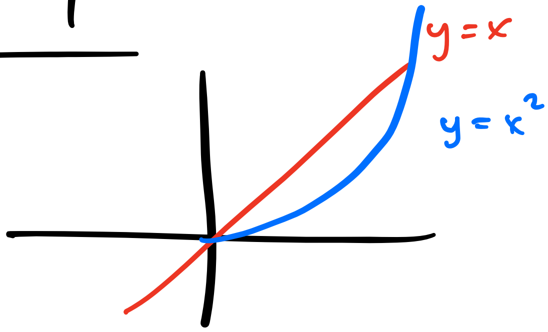
washers about

$\int_a^b \pi (R^2 - r^2)\,dx$

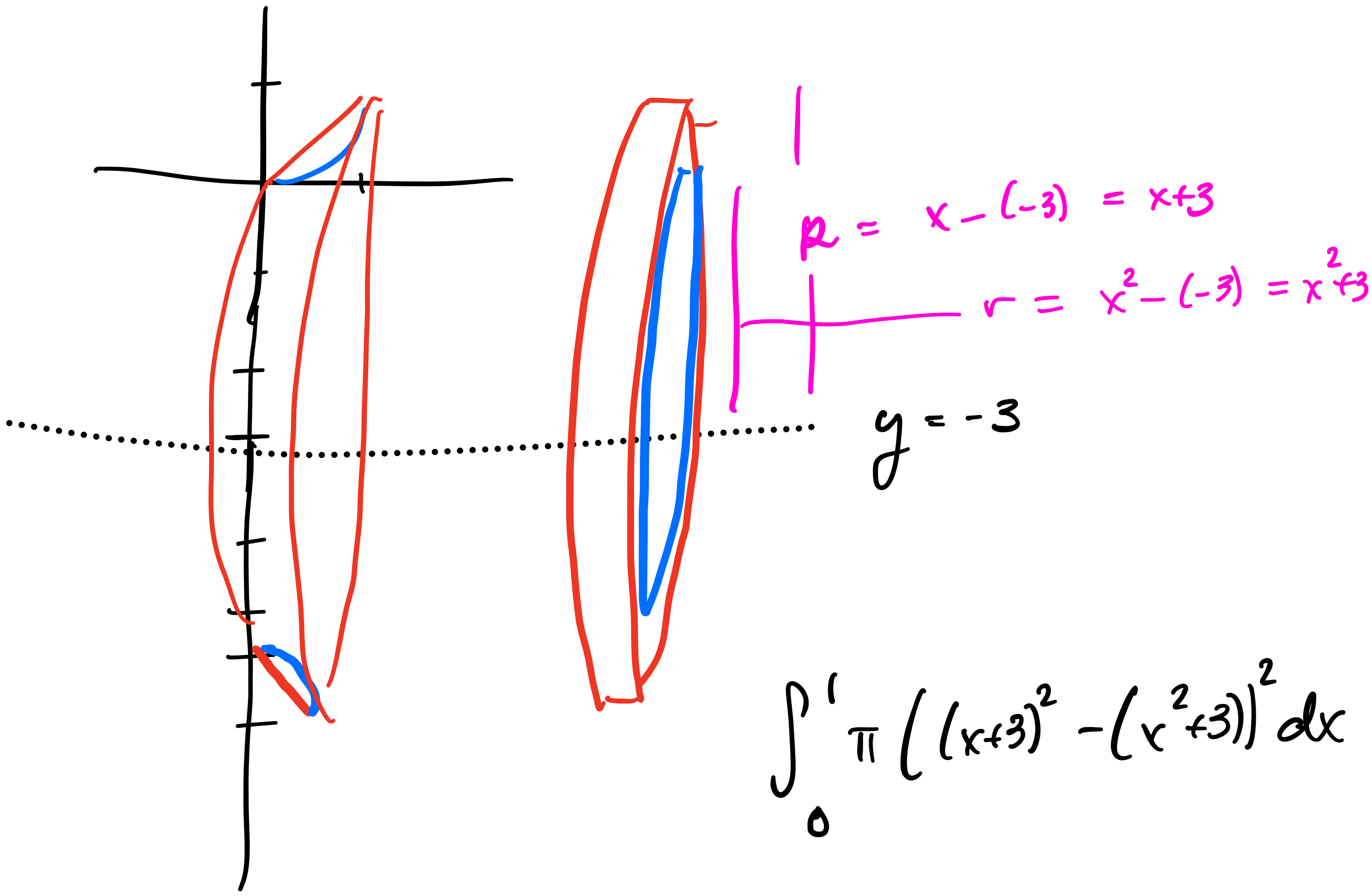
$\int_a^b \pi (R^2 - r^2)\,dy$

Example 4:

rotate



about $y=-3$ and find volume.



$$\int_0^1 \pi\left((x+3)^2-(x^2+3)^2\right)dx$$

$$=\frac{4}{5}\pi.$$