Integration Review

An indefinite integral is another name for an anti-derivative.

Ex: $\int x^2 dx = \frac{1}{3}x^3 + C$

because $\frac{d}{dx}\left(\frac{1}{3}x^3 + C\right) = \frac{1}{3}\cdot 3x^2 + 0$

$= x^2.$

Def: $F(x)$ is the indefinite integral of $f(x)$ if $F'(x) = f(x)$.

If so, we write $\int f(x)dx = F(x)$.

Some integrals you should know:

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C \text{ for } n \neq -1.$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

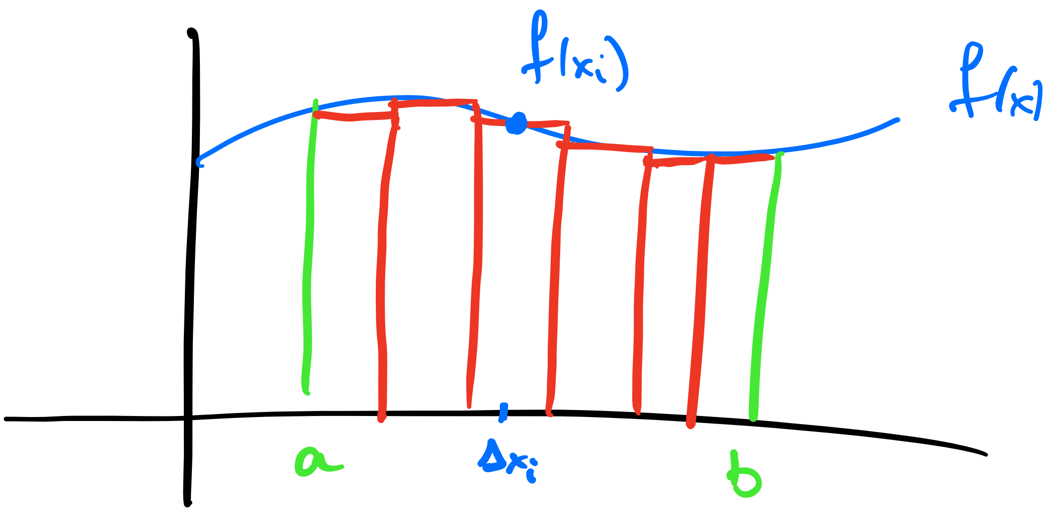
$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

A <u>definite</u> <u>integral</u> is a measurement of (signed) area under a curve.

It is a <u>limit</u> of approximating area by rectangles.



$$A_i = f(x_i)\Delta x$$

$$A \approx \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} f(x_i)\Delta x_i$$

Calculus idea: let $\Delta x \to 0$.

lots of rectangles,
better approximation.

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\Delta x = A = \int_a^b \underbrace{f(x)dx}$$

still the "area" of a rectangle

Fundamental theorem of Calculus

relates these.

$$\int_a^b f(x)dx = F(b) - F(a)$$

area

antiderivatives

all of this should be review. If it isn't, go look at my videos "Introduction to Integration".

Integration is linear

$$\int x^2 + 2x + 3\cos x \, dx$$

$$= \int x^2 dx + 2\int x dx + 3\int \cos x dx$$

$$= \frac{1}{3}x^3 + x^2 + 3\sin x + C$$

what if we have to integrate a product?

$$\int \sec x \tan x dx = \sec x + C$$

because $\frac{d}{dx}\sec x = \sec x \tan x$

what if the product isn't obviously a derivative?

$$\int 2xe^{x^2} dx$$

One place products show up is as a result of the chain rule. Let $F' = f$

$$\frac{d}{dx} F(g(x)) = f(g(x)) \cdot g'(x)$$

so $$\int f(g(x)) g'(x) dx = F(g(x)) + C.$$

we can undo the chain rule with a <u>substitution</u>

$$u = g(x)$$
$$du = g'(x)dx$$

then $\int f(g(x))\, g'(x)dx$

$$= \int f(u)\, du$$
$$= F(u) + C$$
$$= F(g(x)) + C.$$

Strategy: let u = most complicated expression inside another function

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Ex:

$$\int \boxed{2x}\, e^{\boxed{x^2}}\, \boxed{dx}$$

$u = x^2$

$du = 2x\,dx$

$$= \int e^{\boxed{x^2}}(2x\,dx)$$

$$= \int e^u\, du = e^u + C = e^{x^2} + C$$

Ex: $\int \sqrt{2x-1}\,dx$

$u = 2x-1$

$du = 2dx$

$\frac{du}{2} = dx$

$$= \int \sqrt{u}\left(\frac{du}{2}\right)$$

$$= \frac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{3/2} + C$$

$$= \frac{1}{3}u^{3/2} + C$$

$$= \frac{1}{3}(2x-1)^{3/2} + C$$

$\int \frac{x}{(x^2+3)^2}\,dx$

$u = x^2+3$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$= \int \frac{\cancel{x}}{u^2}\left(\frac{du}{2\cancel{x}}\right)$$

$$= \frac{1}{2}\int u^{-2}\,du$$

$$= \frac{1}{2}\frac{u^{-1}}{-1} + C = \frac{-1}{2}u^{-1} + C$$

$$= \frac{-1}{2} \quad \frac{1}{x^2+3} + C$$

Definite integrals.

$$\int_1^2 \frac{dx}{(3-5x)^2}$$

$$= \int_{x=1}^{x=2} \frac{dx}{(3-5x)^2}$$

$u = 3 - 5x$

$du = -5dx \qquad \frac{du}{-5} = dx$

$u(1) = 3 - 5(1) = -2$

$u(2) = 3 - 5(2) = -7$

$$= \int_{u=-2}^{u=-7} \frac{(du/_{-5})}{(u)^2}$$

$$= \frac{-1}{5} \int_{-2}^{-7} \frac{1}{u^2} du$$

$$= \frac{1}{5} \cdot \frac{1}{u} \Big|_{-2}^{-7}$$

$$= \frac{1}{5} \cdot \frac{1}{-7} - \frac{1}{5} \cdot \frac{1}{-2}$$