1 Functions Learned

`PrimeQ`	`Sin`	`Cos`	`Tan`	`Sqrt`	`N`
`Log`	`Exp`	`ArcSin`	`ArcCos`	`ArcTan`	`Simplify`
`FullSimplify`	`RootReduce`	`FactorInteger`	`PrimePi`	`Sum`	`Binomial`
`Mod`	`Quotient`	`Expand`	`Factor`	`Together`	`Apart`
`TrigExpand`	`Random`	`Slider`	`Manipulate`	`Dynamic`	`LCM`
`Fibonacci`	`Divisible`	`First`	`Last`	`Drop`	`Take`
`Rest`	`Most`	`Flatten`	`Range`	`Table`	`Print`
`RandomInteger`	`Map(/@)`	`Select`	`Length`	`IntegerDigits`	`Reverse`
`FromDigits`	`OddQ`	`Divisors`	`Prime`	`Union`	`Product`
`EvenQ`					

2 Problems

From electronic text

1. Problem 1.1 (Check out the function called `RootReduce`.)
2. Exercise 1.1
3. Problem 1.2
4. Problem 1.3 (The book actually computes the probability that a randomly chosen 13-digit number will be prime. Fix it and compute the correct proability.)
5. Exercise 1.2
6. Exercise 1.3 (You need only do this for one number; pick it to be at least 20. Check out the function `Range` and `Apply`.)
7. Exercise 1.4
8. Problem 1.4
9. Problem 1.5
10. Exercise 1.5
11. Exercise 1.6
12. Problem 1.6
13. Exercise 1.7
14. Problem 1.7
15. Problem 1.8
16. Problem 1.9 (Only do this for $1 \leq m, n \leq 13$ instead of $1 \leq m, n \leq 100$ as that's just ridiculous.)
17. Problem 2.1
18. Exercise 2.1
19. Problem 2.2
20. Problem 2.3
21. Exercise 2.2
22. Problem 3.1
23. Problem 3.2
24. Problem 3.3
25. Problem 3.4
26. Problem 3.6
27. Exercise 3.1
28. Problem 3.7
29. Problem 3.8
30. Exercise 3.2
31. Exercise 3.3
32. Problem 3.9
33. Exercise 3.4
34. Exercise 3.5
35. Problem 3.10
36. Problem 3.11
37. Exercise 3.6
38. Exercise 3.7
39. Problem 3.12
40. Exercise 3.8
41. Problem 3.13
42. Problem 3.14
43. Exercise 3.9
44. Exercise 3.10
45. Problem 3.15
46. Problem 3.16
47. Problem 3.17

Additional problems

48. Compute and/or simplify the following.

    (a) $\arctan(2-\sqrt{3})$.

    (b) $\binom{52}{5}$ (the number of possible poker hands.)

    (c) $\tan(\arcsin(x))$.

    (d) $e^{7\ln(3)}$.

49. Does the equality $\tan(x)+\sec(x)=\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)$ hold for all x?

50. Consider the sequence $\{a_n\}_{n=1}^{\infty}$ where $a_n=n^2+n+1$.

    (a) Create a list containing the 121st through 1000th terms of $\{a_n\}$ and store it as a local variable called `myList`.

    (b) What is the 600th term in `myList`?

    (c) Using a function, extract the first and last elements of `myList`.

    (d) Using a function, extract the first 20 elements and the last 20 elements of `myList`.

    (e) Using a function, extract the elements of `myList` that are prime.

    (f) How many elements of `myList` are prime?

    (g) What are the smallest and largest primes in `myList`?

51. Define a sequence $\{h_n\}_{n=1}^{\infty}$ by $h_n=p_1p_2\cdots p_n+1$ where p_i is the i-th prime number. Verify that the factorization of h_n contains a prime larger than p_n for at least 10 different values of n. Based on this evidence, you might suspect that h_n always contains a prime factor larger than p_n for all n. This is in fact true and this construction is used in Euclid's famous proof of the infinitude of primes. Also, the function `Product` may be useful.

52. Define a function called `DigitSum` that sums the digits of an integer and use it to

    (a) sum up the digits of 2^{60}.

    (b) create a list of digit-sums of the first 100 integers.

53. **Using a pure function** that tests whether a particular integer is a palindrome,

    (a) estimate the probability that a random three-digit integer is a palindrome by testing 300 random three-digit integers.

    (b) compute the actual probability that a random three-digit integer is a palindrome.

    (c) compute the error in your estimate from (a).

54. Generate data and conjecture the truth value of the following propositions involving Fibonacci numbers. Here, f_n denotes the n-th Fibonacci number.

    (a) $f_n^2+f_{n+1}^2=f_{2n+1}$.

    (b) there exists is an integer between 0 and 9 that never appears in the ones digit of f_nf_{n+1}.

    (c) $\sum_{i=1}^{n} f_{2i-1}=f_{2n}-1$ (the function Sum may be useful).